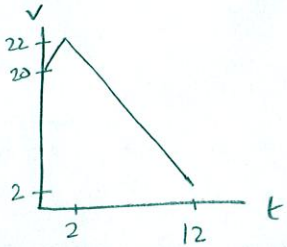
A person's velocity (in meters per second) at time t (in seconds) is given by $v(t) = \begin{cases} 20+t, & 0 \le t \le 2 \\ 26-2t, & 2 \le t \le 12 \end{cases}$.

SCORE: _____ / 5 PTS

[a] Find the exact distance the person travelled from time $t = 0$ seconds to $t = 12$ seconds.
<u>NOTE: You must show the arithmetic expression that you used to get your answer.</u>

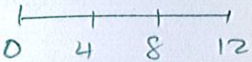


$$\underbrace{\tfrac{1}{2}(2)(20+22)}_{1} + \underbrace{\tfrac{1}{2}(10)(22+2)}_{1}$$

$$= 42 + 120 = \underbrace{162}_{\frac{1}{2}} \text{ m}$$

[b] Estimate the distance the person travelled from time $t = 0$ seconds to $t = 12$ seconds using three subintervals and left endpoints.
<u>NOTE: You must show the arithmetic expression that you used to get your answer.</u>

$$\Delta x = \frac{12-0}{3} = 4$$

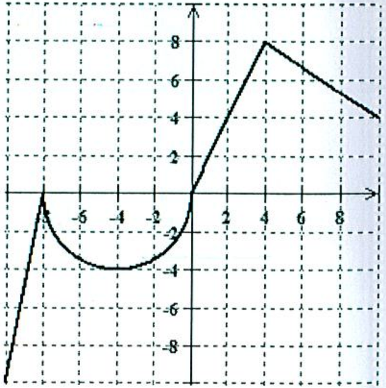


$$f(0)\Delta x + f(4)\Delta x + f(8)\Delta x = \underbrace{20}_{\frac{1}{2}} \cdot 4 + \underbrace{18}_{\frac{1}{2}} \cdot 4 + \underbrace{10}_{\frac{1}{2}} \cdot 4 = \underbrace{192}_{1} \text{ m}$$

The graph of function f is shown on the right.

SCORE: _____ / 4 PTS

The graph consists of a diagonal line, an arc of a circle, then two additional diagonal lines.



[a] Evaluate $\int_{-10}^{10} f(x)\,dx$.

NOTE: You must show the arithmetic expression that you used to get your answer.

$-\frac{1}{2}(2)(10) - \frac{1}{2}\pi(4)^2 + \frac{1}{2}(4)(8) + \frac{1}{2}(6)(8+4)$

$= -10 - 8\pi + 16 + 36$

$= 42 - 8\pi$

$\frac{1}{2}$ POINT EACH EXCEPT AS NOTED

[b] Evaluate $\int_{10}^{-4} f(x)\,dx$.

$= -\int_{-4}^{10} f(x)\,dx = -\left[-\frac{1}{4}\pi(4)^2 + 16 + 36\right] = 4\pi - 52$

1

Using the limit definition of the definite integral, and right endpoints, find $\int_{-4}^{-1} (2x^2 + 8x)\, dx$. **SCORE: _____ / 10 PTS**

NOTE: Solutions using any other method will earn 0 points.

$$\Delta x = \frac{-1 - -4}{n} = \frac{3}{n}$$

2

1 $\lim_{n \to \infty} \sum_{i=1}^{n} \left(2\left(-4 + \frac{3i}{n}\right)^2 + 8\left(-4 + \frac{3i}{n}\right)\right)\frac{3}{n}$ 1

$$= \lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} \left(2\left(16 - \frac{24}{n} i + \frac{9}{n^2} i^2\right) - 32 + \frac{24}{n} i\right)$$

$$= \lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} \left(-\frac{24}{n} i + \frac{18}{n^2} i^2\right)$$ 2

$$= \lim_{n \to \infty} \frac{3}{n} \left(-\frac{24}{n} \sum_{i=1}^{n} i + \frac{18}{n^2} \sum_{i=1}^{n} i^2\right)$$

$$= \lim_{n \to \infty} \frac{3}{\cancel{n}} \left(-\frac{\overset{12}{\cancel{24}}}{n} \frac{n(n+1)}{\cancel{2}} + \frac{\overset{3}{\cancel{18}}}{n^2} \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}\right)$$

1 1

$$= 3(-12 + 6)$$ 1

$$= -18$$

1

+ 1 POINT IF YOU WROTE $\lim_{n \to \infty}$ ON EACH LINE THAT CONTAINS "n"

Evaluate $\int_{-6}^{0} (2\sqrt{36-x^2} - |x+2|)\, dx$ using the properties of definite integrals and interpreting in terms of area. **SCORE: _____ / 5 PTS**

NOTE: You must show the proper use of the properties of the definite integral, NOT just the arithmetic.

2 $\quad 2\int_{-6}^{0} \sqrt{36-x^2}\, dx - \int_{-6}^{0} |x+2|\, dx = 2 \cdot \frac{1}{4}\pi(6)^2 - \left(\frac{1}{2}\cdot 4\cdot 4 + \frac{1}{2}\cdot 2\cdot 2\right)$ 1 1

$= 18\pi - 10$

1